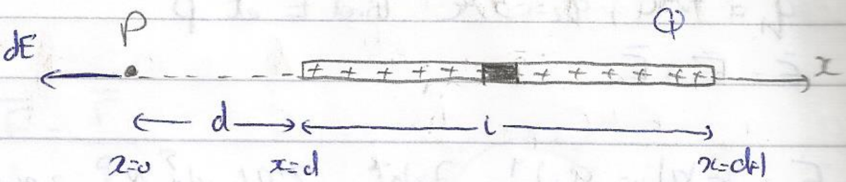
Example

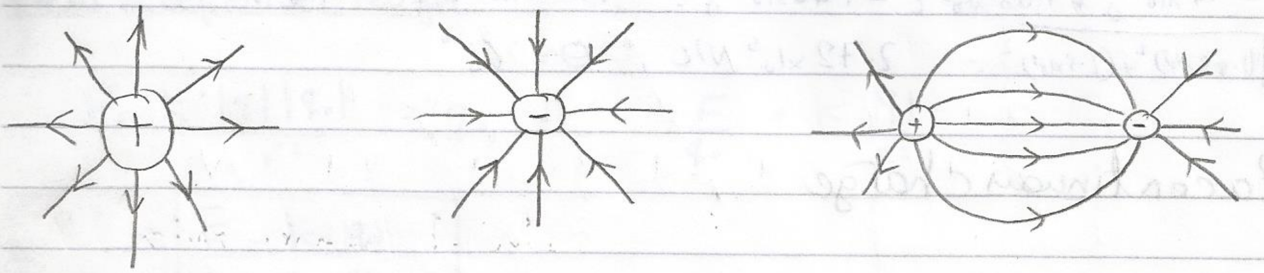


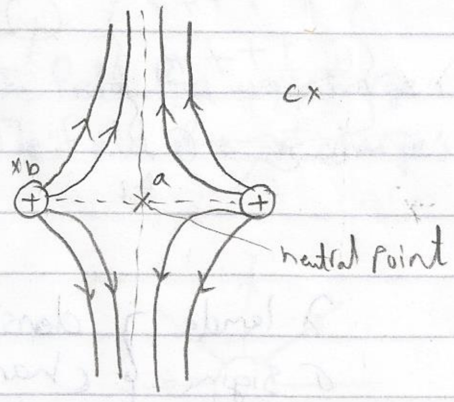
$$dE = k\frac{\lambda dx}{x^2}$$

$$E = k\lambda \int_d^{L+d} x^{-2}\,dx \quad , \quad E = k\lambda \left[\frac{-1}{x} \right]_d^{L+d}$$

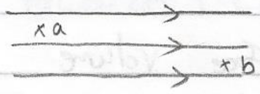
$$= -k\lambda \left(\frac{1}{L+d} - \frac{1}{d} \right) = \frac{k\lambda L}{d(L+d)} \quad , \quad E = \frac{kQ}{d(L+d)}$$

Electric field lines





uniform field



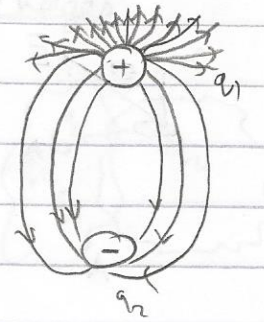
$E_a = E_b$

$E_b > E_c > E_a$

properties

1- Electric field lines do not form closed loops ie it can have a beginning or an end or both
2. Using electric field lines we can determine regions of strong/weak field
3. No intersection between 2 field lines
4- magnitude of charge is directly proportional to electric field lines

Ex)



Find ratio of q_1 to q_2

$$\frac{q_1}{q_2} = \frac{20}{6}$$

Gauss's law

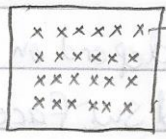
Electric flux (ϕ): It's the no. of electric field lines crossing normally through surface area (A)

a) Flux of uniform field

i) $\bar{E} \perp \bar{A}$

$$\phi = EA$$

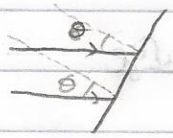
unit: $\frac{N}{C} m^2$



ii) $\bar{E}$ makes angle θ with $\bar{A}$

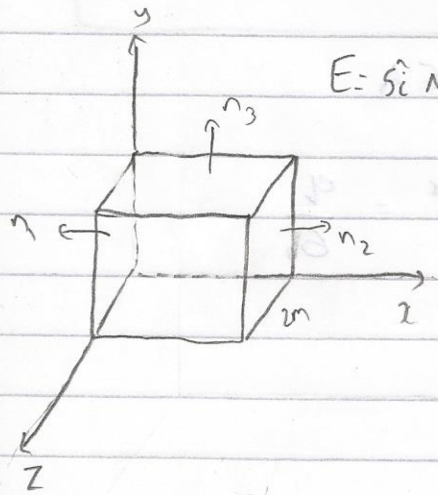
$$\phi = EA\cos\theta$$

$$\phi = \bar{E} \cdot \bar{A}$$



Ex)

$E = 5\hat{i}$ N/C



$\phi_1 = \bar{E} \cdot \bar{A}_1$

$= 5\hat{i} \cdot (4(-\hat{i})) = -20 \frac{N}{C} m^2$

$\phi_2 = \bar{E} \cdot \bar{A}_2$

$= 5\hat{i} \cdot 4\hat{i} = 20 \frac{N}{C} m^2$

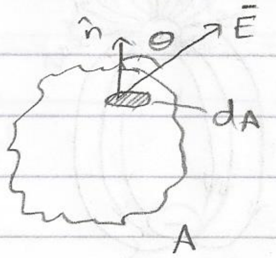
$\phi_3 = \bar{E} \cdot \bar{A}$

$= 5\hat{i} \cdot 4\hat{j} = 0$

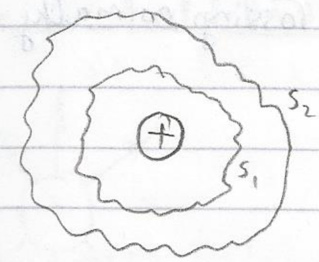
b) Flux of non uniform field

$d\phi = \bar{E} \cdot d\bar{A}$

$\phi = \int \bar{E} \cdot d\bar{A}$



Net flux (ϕ_{net})



$\phi_1 = \phi_2$

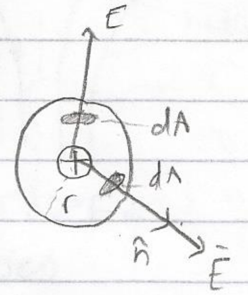
$\phi_{net} = \oint \bar{E} \cdot d\bar{A}$

closed surface

Net flux does not depend on

1- Geometry of closed surface
2- Size of closed surface
3- position of charge in closed surface

Net flux of a point charge



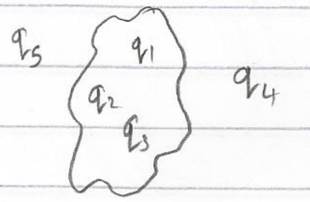
charge in centre $\Rightarrow \theta = 0$

$\phi_{net} = \oint E\, dA$

$= E \oint dA = EA$

$= \left(\frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}\right) \cancel{4\pi r^2} = \frac{q}{\epsilon_0}$

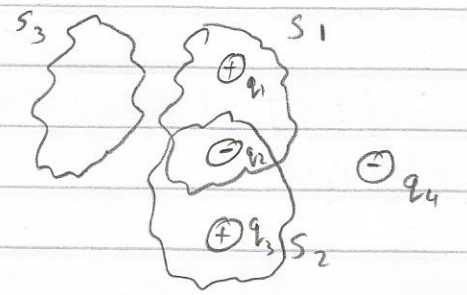
for N-point charges



$$\phi_{net} = \frac{q_1 + q_2 + q_3}{\epsilon_0}$$

* q is used with its charge

Ex) find ϕ_1, ϕ_2, ϕ_3



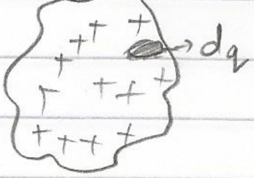
$$\phi_1 = \frac{q_1 - q_2}{\epsilon_0}$$

$$\phi_2 = \frac{q_3 - q_2}{\epsilon_0}$$

$$\phi_3 = 0$$

Net flux of continuous charge

$$\int d\phi = \int \frac{dq}{\epsilon_0} \quad , \quad \phi = \frac{1}{\epsilon_0} \int dq$$



$$\phi = \frac{q}{\epsilon_0} = \frac{q_{inside}}{\epsilon_0} = \frac{\sum q}{\epsilon_0}$$

$$\boxed{\phi_{net} = \oint \vec{E} \cdot d\vec{A} = \frac{q_{inside}}{\epsilon_0}}$$ Gauss's law